Estimations Of Urban Land Use By Fractal And Cellular Automata Method

Mehmet Ali YUZER

Dr. Mehmet Ali YUZER, Department of City and Regional Planning; Istanbul Technical University, İstanbul, Turkey; E-mail: syuzer@turk.net

1 INTRODUCTION

The article consists of five parts. After the introduction part the evolution of land use models and cellular automata based models are explained. Later, fractal and cellular automata concepts and running principles of a cellular automata system are expressed. In the other part the main properties of LUCAM model, the simulation of the model to Bursa city and conditions after this simulation are defined. In the last part after explaining the guidance effect of the model in planning studies suggestions are expressed.

2 EVOLUTION OF LAND USE MODELS AND CELLULAR AUTOMATA (CA) BASED LAND USE MODELS

In 1882, the ideas of Soria Y. Mata paved the way for new searches for replanning of settlements in accordance with certain principles. Following this there was a progress of development by means of Linear Urban Forms created by Tony Garnier, a French architect in 1917, by Milyutin in 1929, by Hilberseimer in 1941 and by J. Jose Sert in 1948. During these periods, there were a variety of arguments and models about not only Linear Urban Forms but also radiatic and radio concentric urban forms (Morris 1979). In 1919 it was Paul Wolf who developed radio concentric urban form to be used as an alternative for linear urban form. This study was followed by a model designed by A. Edwards in 1930s, which separated the functional areas by means of radiatic roads originating from the city center, and which kept the small working areas in the center whereas the big ones out of the settlement and which had radiatic as well as radioconcantric features. In 1946, S.E. Sanders and A.J. Rabuck developed a model, which was planned in a way to make the working areas, and their concerning settlement regions exist together in a concentric settlement. Following these models there were various approaches to settlement planning in different periods of time. All these searches that started with S. Mata's in 1882 were based upon accessability and the relationship between the function of work and that of housing due to the importance of production (Mumford 1961).

In the mid-1900s, although computers were not used in the models developed for estimations about the ways to use the locations, there were a considerable number of studies in which mathematical concepts were densely used. In the studies carried out by economists, the assumptions and models were rather made with sizes that could be taken as samples or by using calculators. However the common feature of them was that they were designed with scales taking the ability of the human factor into consideration. It was especially after 1980s that the studies about the models used to estimate the progress of land use increased because of the fact that computers were transformed into personal ones, became widespread and operations were fulfilled in a shorter period of time because of technological developments.

As computers were widely used in all kinds of disciplines, the data exchange between the discipline of City Planning and other disciplines (geography, photographmetry and environmental sciences) and as a result it was possible to combine the data of all these disciplines in a short time to make more proper estimations about land use. Towards the late 1980s, in the discipline of geography, computer programs, which were able to save the data swiftly, make input and output operations among the data, search and assess them according to the given target, were developed and by this way it was possible to make simulations concerning land use (Peuquet, Marble 1990; Archer v.d. 1961).

Before mentioning the CA based model used to make estimations about the land use (LUCAM), which is the main subject of this article, it is significant to present a historical trace back of the studies made so far, which will aid the reader to understand the relationships used in the method.

2.1 Models That Determine the Area of Effect

Since 1960s there have been efforts to apply various methods based on computers and urban simulations in order to estimate the growth processes and to set the orientations of the growth.

The first attempt in modelling studies carried out to estimate the land use was the studies done before 1960s to plan the transportation in metropolitan areas with the aid of computers. During the development processes of Detroit Urban Simulation Model, Metropolitan Area Transportation Model (1955-1956) and Transportation Model of Chicago, the modelling studies, which had been made before, were used to a great extent. The Urban Simulation Model developed by NBER (National Bureau of Economic Research) is a generalized model based on empirical researches about a great number of cities and it has had political influences. This model is known to be the first of the computer simulations on urban growth and development (Kain, 1975). The model was designed to simulate the main changes in urban special structure caused by the future effects of employment and population growth, income increase, changes in transportation technology (Gregory, Kain and Ginn, 1972). Another study carried out for Detroit is called Detroit Urban Transportation Planning. In this study it was accepted that the number of travels depends on the demands caused by land use whereas the future travels can be set in accordance with the demands stemming from estimations about land use. Taking the weaknesses of Detroit into consideration, a model that presented the relationship between land use and transportation was developed for Chicago later on. In this model called Chicago Area Transportation Study (CATS) it was found out that by using computers the speed of transportation increased in the metropolitan area and it took less time to travel from one place to another (Gregory, Kain, Ginn 1972).

2.2 Econometric Models

In 1960s, a prototype model based on linear programming was developed by Herbert-Stevens to estimate the growth of settlement areas. The use of this model in theoretical and empirical researches led to an innovation in urban economy-wise planning (Kain, 1975,1987). In the mid 1960s, within Boston Regional Transportation Study an empirical model was developed in order to describe the changes in the rates of income, population and employment. This model was used to estimate the distribution of population and employment in spaces. The model was applied to many settlements such as Atlanta, Denver and Washington.

2.3 Pull-Type Models

The pull-type models developed for the estimation of land use trace back to the study of Ira S. Lowry published with the title of “Lowry Model” in 1961. This model was widely used in Great Britain and other European countries. Since the main industrial area had already been located in a certain area, the model was designed for the purpose of making estimations about the possible residential areas of the population that would work in and serve for the concerning sector and the transportation network among the pertinent districts (Lowry 1964). Lowry model was used as the core study of several functional estimation studies i.e. “Time Oriented Metropolitan Model” (TOMM) by Crecine (1964), “Planned Land Use Model” (PLUM) by Goldner (1968) and that of Putman (1976).

Batty (1971), an English scientist, published studies that described the differences between the models with sub-region scales and those with urban scales. The Nottingham–Derbyshire model developed by Batty on the basis of these differences is accepted to be the first effort to estimate the pull-type models on a sub-region scale. Batty (1973) developed his first demand model by integrating a pull type model into a spatial distribution model and at the end created a housing market model. As a result of this study called Reading Model, Batty developed two new versions of simple demand model by making use of the data about the election of work places and settlements (Batty, 1976).

2.4 CA Based Land Use Simulation Models

In the studies made for urban systems many Cellular Automata based models have been developed. Tobler, who carried out a study in 1979, is the first person who used cellular approach in geographical planning and his studies were followed by the studies of Couclelis published in 1985, 1988, 1989, 1996 and the study of Takeyama made in 1996. The studies of Coucleis (1996) and Takeyama (1996) had a generalized modelling language enabling integrated, dynamic and spatial modelling on all kinds of scales, on the basis of GIS. Batty and Xie (1996) developed a CA based model for not only the land use samples but also the urban modelling with integrated transportation network (Kain, 1987).

Portugali ve Benenson made researches about the general organization principles of the city with the aid of CA models within the framework of the studies carried out in 1995 and 1997. The above-mentioned researches were based on the social structure of the city. In those studies emphasizing a model in which the characteristics of social structure in cellular location were described, the cellular space represented the individual housing parcels and the cellular conditions reflected the characteristics of the family who live in the cell as well as the criteria such as the value and the status of the residence (Portugali, Benenson, 1995).

The studies about the application of CA on urban structure made by Cecchini ve Viola in 1990 and 1992 and by Cecchini in 1996 included a model made by means of simple automata. Cecchini developed a model which comprised cellular-type of relations among neighbours and which was based on physical data including density in order to design the urban form and he used this model to simulate the growth of an urbanized area (Cecchini, Besussi, 1996).

In the researches on CA based urban models made by White and Ebgelen (1993, 1994, 1997) and by White (1997) and by Engelen (1997), C.A. based urban models, which combined the theoretical concepts with the ampirical facts, were developed (White R., Engelen G., 1997-b) In those C.A. based urban models founded upon high sensitivity analysis, it was agreed that the settlements had four main functions (trade, industrial, residential and empty areas) and that there is a pulling or pushing power among the cells that may vary according to distances. The objectives of the research were estimating and simulating the land use of the settlements according to the relationships among cells. The main difference between these models developed by White and Engelen and the models based on socio-economic events and transportation network is that White and Engelen’s is a model based on land use (White, Engelen, 1993, 1994)

3 FRACTAL, CELLULAR AUTOMATA CONCEPTS AND THEIR RELATIONSHIP WITH URBAN PLANNING

When the development process of several events in natural, physical and social environment are examined, in differentations seen in initial and the next time interval, it’s seen that initial position affects the next position much. Especially functional changes in existing texture (such as commerce, service functions) shows it’s effects in a short time and cause differentations in texture. The dimension of this differentation changes according to the function’s quality and effect. Mostly industrial enterprise occurs in a cell unit cause the development of other industrial enterprises, settlement areas, commerce and other service functions.

3.1 The Concept Of Fractal

So as to explain the relation between fractal structure and nature, several phenomena can be examined. Researces, which has done to explain the relation of natural phenomena with fractal method, show that distribution of earthquakes follows a special mathematical pattern and it constitutes similar properties with distribution with distribution of incomes per capita in a liberal economy. All the blood vessels from aorta to capillary vessels forming another type of continuous series can be examined. Vessels bifurcate and divide then again bifurcate and so on. The nature of the vessel's bifurcation is fractal. The fractal system can also be seen in the structure of a tree or bone system of a man or transportation system of a settlement. Golden ratio has been used for a long time by specialists like architects and city planners. In golden ratio by the iteration of rectangles in definite ratios a spiral similar to the initial rectangle occurs. Mathematical comment of infinite and self similar iterations occurs fractal geometry.

Methods using cellular automata mechanism for land use estimation depends on the fractal structure in settlements have been developed. The model suggested in the article is also a method, which developed through cellular automata (cellular transformation) rules. Urban land use estimation simulation model with cellular automata, which developed by G. Engelen, R. White contributed to the development of suggested model of the research.

3.2 Cellular Automata-Automation System-Rules

Sierpinski accepted cellular automata as the initial point of science. In a way Pascal triangle was the first cellular automata. Studies done by Konrad Zuse, Stonislaw Ulam, John Van newmann about cellular automata was related with the production of the first counting machines. Nowadays cellular automata has been used in so many science and technology from physic, chemistry, biology to countable liquid dynamics in aeroplane and ship design, from philosophy and sociology to geography and city planning as an important modelling and simulation means.

- A cellular automata is characterised by these main properties (Rietman, 1992).
- It forms from regular cell lattice.
- Development occurs in time steps.
- Every cell characterise with a situation.
- Every cell develops by the same rule only depend on the situation of the cell and neighbour cell number.
- Neighbour relationship is local and self similar.

Two main concepts and their relation is important in the running system of models depend on cellular automata. These concepts are induction and deduction approaches between real world (world of observed events-events to be modelled) and certain world (modelled events world) (Colonna, Stefano, Lombardo, Papini, Rabino, 1998).

A cellular automation system is modelled in three conditions.

Rest (0), Stimulate (2) and Back to normal (1). Cell development is characterized by rules as shown down. If a cell isn't stimulated by another neighbour cell, this cell will be in rest. If a cell is stimulated by at least one neighbour cell, this cell will be stimulated and named a "stimulated cell". A cell which is stimulated transform a normal cell after one step further. A cell which is transformed normal condition, transforms rest position.

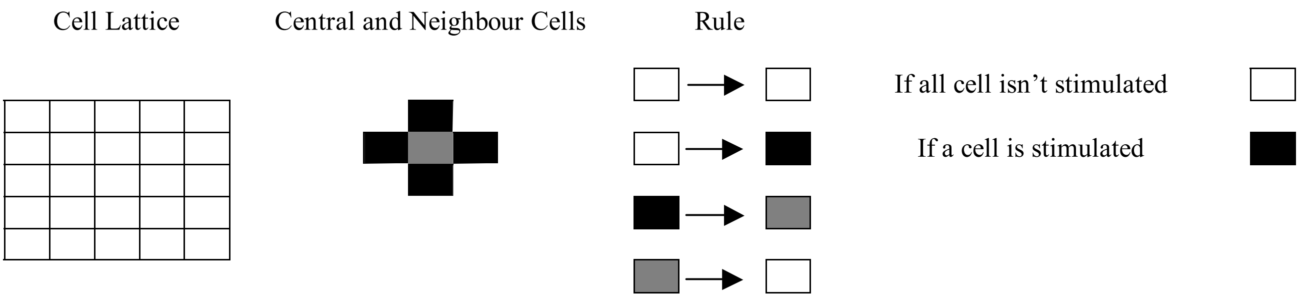

Figure 1: The Example Schema That Cellular Automata Rules are Applied (Weimar, 2000).

0->0 If a cell isn't stimulated by another neighbour cell, (kh≠2)
0->2 If a cell is stimulated by a neighbour cell (kh=2)
2->1; A cell which was stimulated, it transforms normal condition.
1->0; A cell, which is transformed normal condition, it is transform rest condition.

4 CELLULAR AUTOMATA BASED MODEL OF LUCAM

The objective of this model developed for estimations about land use in urban sites is to obtain the data, which will contribute to the formation of growth orientations, on the basis of the data about the land use and the wants of the user in the synthesis phase of planning studies. Development of this model (LUCAM) has been founded upon CA developed by G. Engelen and R. White and the simulation model for estimations about settlement land use (Engelen, White, Uljee, 1997; White, Engelen, 1997-a, White, Engelen, 1993-b). The main features and the fiction of the model is based on the method developed by Engelen and White. On the other hand original ways have been followed in data formation and assessment methods. This method has been employed to test the model in Bursa settlement and to evaluate the results. In this model each cell represents a kind of urban land use (residence, industry, trade, facility, vacant. The main features, the operation system and assessment system of this cellular automata based model are mentioned below.

4.1 The Operation System of the LUCAM Model

Operation System 1: The automation is divided into 100x100 m grilled shaped cells. Each cell must be in one of the pre-determined conditions i.e. Vacant (V), Residence (R), Industry (I), Trade (T), Facility (F) . The size of the grill changes according to the size of the settlement in which the model is tested. The area size can be accepted as about 1ha.
Operation System 2: The neighbour unit of a cell is defined as all the cells that remain in the 6 cell distance from the main cell. The total number of the cells that exist in the area between the core cell and the neighbour cells in the sixth zone is 168. As the cells are in a regular order, every cell in the neighbourhood is found in a 100 meter distance band (zone 1) or 848 meter distance band (zone 6).
Operation System 3: According to the area values and economic effects the hierarchy of the lowest and highest conditions in the land use model should be as follows:
In this hierarchy an empty cell can be transformed into any other function but an industrial cell can only be transformed into a trade or service function. As we acknowledge that the cities may grow, there can't be a transformation from the highest to the lowest condition (White, Engelen, 1993-a; White, Engelen, 1997-b; Yüzer 2001).

The lowest condition
↓
Vacant Areas
↓
Residential Areas
↓
Industrial Areas
↓
Commercial Areas
↓
Facility Areas
↓
The highest condition

Figure 2: Hierarchy of Urban Land Use Area (White, Engelen, 1994; Yüzer, 2001).

Operation System 4: In each turn, the transformation potentials are calculated for all possible transformations. For one cell the transformation potentials are calculated as a total (Formula 1).

The Calculation Method of the Cellular Transformation Potential

$$\mathbf{P_{ij} = (\textstyle\sum_{k,d} m_{kd}) / \sum h} \qquad (1)$$

$\mathbf{P_{ij}}$ **:** is the transition potential from state i to state j
$\mathbf{m_{kd}}$ **:** is the weighting parameter applied cells in state k in distance zone d
$\mathbf{\sum h}$ **:** is the some of central cell and neigbour cells

Empty cells have no weight and they directly contribute to the transformation potential. The facility areas exist in the settlements as unchanging function areas. These areas are for public use and they can not be transformed into functions such as residence, trade or industrial areas unless there is an extraordinary condition. On the other hand the transformation of facility areas take place if there is a need for another facility for public use. For this reason the cells defined as facility enter the model as they are, do not have any impacts on other cells and do not have any kinds of transformation.

Table 1 The Weighting Parameter to be Applied on Each Cell to Calculate the Transformation Potential (White, Engelen, 1994; Yüzer, 2001).

		Vacant ↓ Commerce				Vacant ↓ Industry				Vacant ↓ Housing				Industry ↓ Commerce				Industry ↓ Industry				Industry ↓ Housing				Housing ↓ Commerce				Housing ↓ Industry				Housing ↓ Housing			
		Commerce	Industry	Housing	Facility	Commerce	Industry	Housing	Facility	Commerce	Industry	Housing	Facility	Commerce	Industry	Housing	Facility	Commerce	Industry	Housing	Facility	Commerce	Industry	Housing	Facility	Commerce	Industry	Housing	Facility	Commerce	Industry	Housing	Facility	Commerce	Industry	Housing	Facility
Distance to central cell (d)	1	0,8	0,0	0,3	-	0,0	0,8	0,0	-	0,0	0,0	0,3	-	0,2	0,0	0,0	-	0,0	0,0	0,0	-	0,0	0,0	0,0	-	0,1	0,0	0,05	-	0,0	0,05	0,0	-	0,0	0,0	0,0	-
	2	0,8	0,0	0,3	-	0,0	0,8	0,0	-	0,0	0,0	0,3	-	0,1	0,0	0,0	-	0,0	0,0	0,0	-	0,0	0,0	0,0	-	0,1	0,0	0,05	-	0,0	0,05	0,0	-	0,0	0,0	0,0	-
	3	0,8	0,0	0,3	-	0,0	0,8	0,0	-	0,0	0,0	0,3	-	0,1	0,0	0,0	-	0,0	0,0	0,0	-	0,0	0,0	0,0	-	0,1	0,0	0,02	-	0,0	0,04	0,0	-	0,0	0,0	0,0	-
	4	0,6	0,0	0,2	-	0,0	0,6	0,1	-	0,1	0,02	0,2	-	0,5	0,0	0,02	-	0,0	0,0	0,0	-	0,0	0,0	0,0	-	0,05	0,0	0,02	-	0,0	0,03	0,01	-	0,0	0,0	0,0	-
	5	0,6	0,0	0,2	-	0,0	0,6	0,2	-	0,2	0,03	0,2	-	0,5	0,0	0,03	-	0,0	0,0	0,0	-	0,0	0,0	0,0	-	0,02	0,0	0,01	-	0,01	0,02	0,02	-	0,0	0,0	0,0	-
	6	0,4	0,0	0,1	-	0,0	0,4	0,3	-	0,3	0,05	0,1	-	0,2	0,0	0,05	-	0,0	0,0	0,0	-	0,0	0,0	0,0	-	0,02	0,0	0,01	-	0,02	0,01	0,05	-	0,0	0,0	0,0	-

In Table 1 each four-column block between the spaces comprise the parameters used in the model. In each block of the table, the first column illustrates the weights applied on trade cells at 1-6 zones, the second the weights applied on industrial cells, and the third the weights applied on residence cells.

In each turn a sufficient number of cells are transformed into all kinds of uses in accordance with the above given hierarchical order. The cell which tests all the statuses, is at the highest potential as long as the conditional hierarchy is suitable for such kind of a case. In other words the end-condition of the cell which tests all the statuses shows the highest possible potential of that cell. The operation is applied on all cells and by this way the scheme following transformation is drawn up. (White, Engelen, 1994). This scheme created by using transformation parameters is operated for the second time to form a basis for the model in case the projected population or the settlement capacity is not achieved. The model is operated until the set size is achieved.

5 CASE STUDY OF MODEL IN BURSA SETTLEMENT

Bursa (Turkey) was decided to be the sample area to test the model. It is one of the metropolitan cities of Turkey and it's population is 1.184.144 according to general census of 2000. Bursa is Turkey's fourth metropolitan city in population growth hierarchy. Within the scope of this research the projected population of Bursa in 2020 was estimated to be 2.813.394 (DIE, 1997, 2002). The primary sectors in Bursa settlement are agriculture, industry and service sectors. There is a rapid growth in the settlement due to the dynamics of development. Because of the rapid population growth in the settlement, there is a growing need for new areas.

In order to set the values of weighting parameters applied on the cells within the model, a research was made to determine the inclinations of the users in 10 different regions consisting of residence, industry and trade areas within Bursa central area and the achieved results were assessed (Yüzer 2001). The settlement was divided into 1 ha areas on the basis of land use to test the model in overall settlement of Bursa (BBBKBSDB 2000). Taking the weighted uses into account, each cell in the settlement divided into 1ha areas was formed according to 5 main functions (trade and service, residence, industry, facility and empty space) (Figure 3).

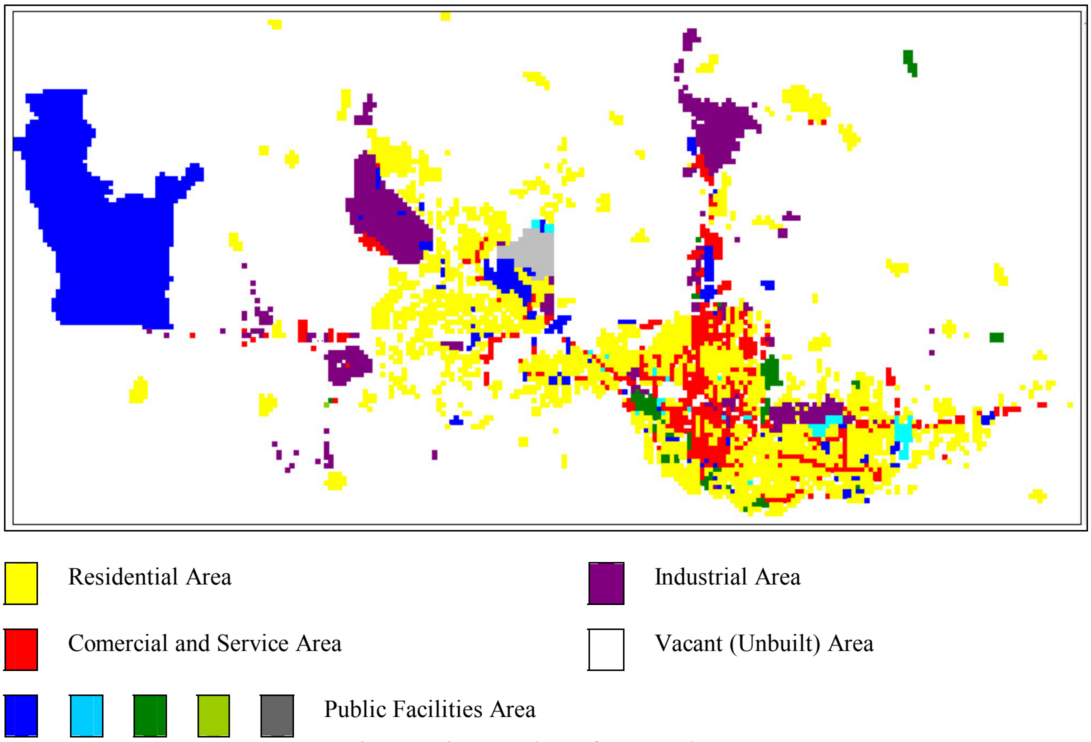

Figure 3 Existent Land Use of Bursa Settlement

According to existing data, functional areas and sizes are determined as follows in Bursa settlement area cellular land use schema (Table 2).

Table 2 Land Use of Bursa Settlement (Existing Situation) (DIE , 1997, 2002; BNİPR, 1995)

	Area (ha)	**(m^2/persan)**
Residential Area	2531	25.3
Comercial and Service Area	660	6.6
Industrial Area	749	7.5
Public Facility Area	1759	17.5
Overall Settlement Area	5699	56.9
Vacant Area	23447	234.4
Total Area (ha)	**29146**	**291.3**

Bursa Settlement After Transformation

The model was operated to find the area need of Bursa settlement for the year 2020 and the values in Table 3 were obtained as a result of the first transformation. In the first transformation process the number of residence cells increased 1.06 times and the end-size was 5228 ha. Moreover the trade cells increased 2.01 times and the industrial cells 2.14 times (Figure 4). Except the facility areas the ratio of the general increase in cells was 2.43 as a result of the first transformation and the end-size of the settlement was 9576 ha. In compliance with the principles of the model, the facility areas did not have transformations of any kind and their number and position remained the same (Yüzer 2001).

Table 3 Land Use of Bursa Settlement After First Transformation (DIE, 1997, 2002; BNİPR, 1995)

	Bursa Settlement Area Distribution After Transformation
Population (person)	2.813.394
Residential Area (ha)	5228
Commercial and Service Area (ha)	1993
Industrial Area (ha)	2355
Public Facility Area (ha)	1759
Overall Settlement Area (ha)	11335
Vacant Area (ha)	17811
Total Area (ha)	**29146**

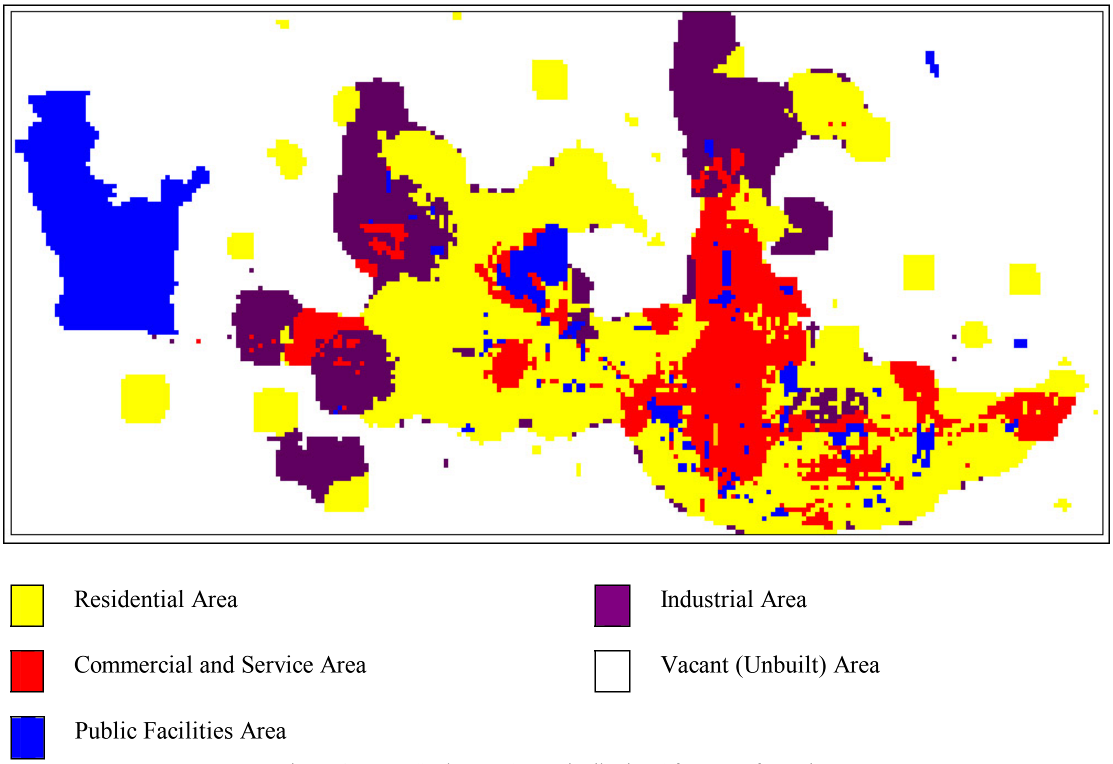

Figure 4. Bursa Settlement Area Distribution After Transformation

Accepting that all the function areas are adequate for Bursa settlement, it was found out that the required area size should be 15958 ha in 2020 on the basis of population growth. This refers to a 2,8 times increase. The difference between the size obtained after the transformation process and the required area size in 2020 was found to be 4622 ha. In this case the model could have been reoperated. However as it was agreed that there would be a growth in facility areas as well, one-stage operation of the model was deemed suitable.

6 CONCLUSION

Fractal and cellular automata based LUCAM Model which is explained in the article estimates land use transformations and new development regions in the settlements. As a result of the evaluations and the application of the model to Bursa it can be accepted that the model is an important guidance in urban planning studies. Easily application of the model to settlements, which are turned to fractal schema, strengthens spatial scale suggestions. Depend on the defined size of the settlement; the running of the model increases the sensibility of both the cell and the model. The model concludes peculiar to the settlement by using variation depend on the condition of the cell as the main rules of the cellular automata. With different rules and parameters put forward in the model enables the development of different versions of the model by different researchers. The model can be organized in different forms depend on local properties. By defining the user's tendencies, differentiations in rules used in the model and weight parameters can be occurred. Thus the model can be run special to that settlement depends on every settlement's property.

In this research, getting population projections did the simulation of Bursa settlement and land use data depend on user's tendencies. In another version of the model, the simulation of the settlement can be done by rules and acceptances determined after using natural data. The model intuitionally put forward logical results. With this characteristic, it guides zoning studies in classical planning techniques. The model enables the transfer of criteria determined in the settlement to cellular schema and getting schema of transformation in harmony with criteria. This model can be integrated with Urban Information System and Geographical Information System and data of these systems can be used in the model.

The model minimizes the calculation time with the software developed in the computer. Furthermore the variability of the matrix size determined depends on the settlement's size enables easy use of the model for every size of the settlement. In the model ownership was ignored. So the results should be compared with the ownership texture. In another version of the model, simulation can be done with data such as ownership texture and land values.

7 REFERENCES

Archer H. And Others.: Geographic Information Systems, Capman&Hall Press, New York 1961

Batty M.: Urban Modelling, Cambridge University Press, UK, 1976

BBBKBSDB.: Bursa Büyükşehir Belediyesi, Kent Bilgi Sistemi Daire Başkanlığı, Bursa 2000

BNİPR.: Bursa Nazım İmar Planı Revizyonu, Bursa Büyükşehir Belediyesi, Bursa, 1995

Cecchini, A.: Besussi, E.: Aproaching Generalized Urban Automata with Help! on Line AUGH, Artificial World and Urban Studies, Venice: DAEST, 1996, pp. 231-248

Colonna A.: Stefano V.D.: Lombardo S.T.: Papini L. Rabino G.A.: Learning Automata for Urban Development Exploration The Case Study of Rome Urban System, 38th ERSA Congress, Vienna, 1998

DIE.: General Population Census Data of Turkey 2000 Transient Data, Ankara, 2002

DIE.: General Population Census Data of Turkey 1997, Ankara 1998

Engelen G.: White R.: Uljee Inge.: Integrating Constrained C.A. Models, GIS and decision Support Tools for Urban Planning and Policy-Making, Decision Support Systems in Urban Planning, Edt. Timmermans H.: E&FN SPON, London, 1997

Gregory K., Kain J.F. and Ginn J.R.I.: The Detroit Protetype of the NBER Urban Simulation Model, Distributed by Columbia University Press, New York.: 1972

Kain J.F.: Essays on Urban Spatial Structure, Ballinger Publishing Company, USA, 1975

Kain J.F.: Computer Simulation Models of Urban Location, Handbook of Regional and Urban Economics, Volume II, Chapter 21, pp. 847-875, Mills E.S.: Elsevier Science Publishers, Amsterdam, 1987

Lowry İ.S.: A Model of Metropolis, The Rand Coorparation, California, 1964

Morris A.E. J.: History of Urban Form: Before the Industrial Revolutions, Longmans Scientific and Technical, London, 1979

Mumford L.: The City in History, Secker&Worburg, London, 1961

Portugali J., Benenson I.: Artificial Planning Experience by Means of a Heuristic Cell-Space Model: Simulating International Migration in the Urban Process, Environment and Planning A 27, 1647-1665, 1995

Peuquet D.J., Marble F.D.: Introductory Readings in Geographic Information Systems, New-York, 1990,

Rietman E.: Creating Artificial Life Self Organization, Windcrest, 1992

White R., Engelen, G.: Cellular Dynamics and GIS: Modelling Spatial Complexity, Geographical Systems 1, 1993-a, Page 237-253

White R.: Engelen, G.: Fractal Urban Land Use Patterns: A Cellular Automata Approach, Environment and Planning A 25, Page 1175-1199, 1993-b

White R., Engelen, G.: Urban Systems Dynamics and Cellular Automata: Fractal Structures Between Order and Chaos. Chaos, Solitons, and Fractals 4, Page 563-583, 1994

White R., Engelen G.: Cellular Automata as the Basis of Integrated Dynamic Regional Modelling, Environment and Planning B 24, Planning and Desing, 1997-a, Pages 235-246

White R., Engelen G.: Multi-Scale Spatial Modelling of Self-Organizing Urban Systems, F. And Haken, H. eds.: Self Organization of Complex Structures: From Individual to Collective Dynamics; Gordon and Brech, 1997-b, Pages 519-353

Weimar J.: Simulation with Cellular Automata, http://www.tu-bs.de/institute/WiR/weimar/Zascriptnew/intro.html

Yüzer M.A.: Şehirsel Yerleşmelerde Fractal ve Hücresel Otomata Yöntemi İle Gelişme Alanlarının Belirlenmesi, İTÜ Fen Bilimleri Enstitüsü, Doktora Tezi, İstanbul, 2001,